**From:** [Charlesworth, Henry](mailto:H.Charlesworth@warwick.ac.uk)  
**Sent:** 03 January 2020 20:34  
**To:** [Reed, Alex](mailto:Alexander.Reed@warwick.ac.uk)  
**Subject:** Re: Meeting next term

That sounds good to me, so meet in the lobby just after 1! See you then!

Henry

**From:** Reed, Alex <Alexander.Reed@warwick.ac.uk>  
**Sent:** 03 January 2020 20:33  
**To:** Charlesworth, Henry <H.Charlesworth@warwick.ac.uk>  
**Subject:** RE: Meeting next term

Hi Dr Charlesworth,

Either is good for me – I’m happy to do it in the maths department but I’m not able to book a specific room so it might be a case of meeting in the lobby and finding somewhere together! Failing that we could then try the IMC building?

Kind regards,

Alex

**From:** [Charlesworth, Henry](mailto:H.Charlesworth@warwick.ac.uk)  
**Sent:** 03 January 2020 20:06  
**To:** [Reed, Alex](mailto:Alexander.Reed@warwick.ac.uk)  
**Subject:** Re: Meeting next term

Hi Alex,

Sorry for the delayed reply! I'm not sure where would be best really, my office is actually not very convenient as I share it with other postdocs. Maybe somewhere in the maths department would be easiest, there are quite a few areas that are usually relatively quiet right? Or if not then I'm based in the WMG IMC building, it would probably be possible to find somewhere quiet there. Whichever you prefer really!

Best,

Henry



**From:** Reed, Alex <Alexander.Reed@warwick.ac.uk>  
**Sent:** 31 December 2019 15:00  
**To:** Charlesworth, Henry <H.Charlesworth@warwick.ac.uk>  
**Subject:** RE: Meeting next term

Hi Dr Charlesworth,

Thank you for answering those questions. I’ll need to look into the correlation calculation a bit more but your explanation is very helpful!

Tuesday 7th any time after 1pm is fine for me. Is there anywhere in particular you would prefer to meet?

Kind regards,

Alex



**From:** Charlesworth, Henry <H.Charlesworth@warwick.ac.uk>  
**Sent:** Tuesday, December 31, 2019 12:00:38 PM  
**To:** Reed, Alex <Alexander.Reed@warwick.ac.uk>  
**Subject:** Re: Meeting next term

Hi Alex,

Thanks, I've had a very nice Christmas, I hope you have too!

I'll be happy to answer any questions you have, and it's no problem if you want to record the conversation! I will be around on campus from next week and have a pretty free calendar I believe, so I think any day when you finish lectures would work for me. If you have nom preference shall we say just after 1 on Tuesday?

Regarding the attached questions:

(1) In the discrete version of the model where you're just counting visual states this is possible, although it doesn't happen too often in practice. When it does I believe the agent just chooses one of the actions with maximal visual states at random.

(2) Yes, this is an annoying typo which has been pointed out before, but now it's been published we can't change it! Good spot though, it should say four nodes.

(3) You could define it in a number of different ways (which won't necessarily be exactly the same, but which should be fairly similar) - we define it in terms of the average opacity that each agent sees itself because I think this is the simplest way.

(4) I think this notation is pretty confusing actually. Fundamentally the correlation length is just <u\_i(0) . u\_j(|r|) >, i.e. the average correlation between two agent's velocities (relative to the COM) when the agents are separated by a distance of r (with optional normalisation). What that notation is trying to convey is the empirical correlation length - if you collect a load of real data then technically you're only able to define the correlation length at the exact distances where you have data - and often you might only actually have one data point at that exact distance. This is not at all useful, and so in reality what you do is smooth this out a bit - most simply by dividing r into bins of finite size dr. We then integrate the empirical correlation function over the bin - so the delta function picks out all of the pairs with distances inside the bin r -> r+dr (and the denominator counts them) - so you end up just calculating the average of u\_i.u\_j for pairs within r and r+dr. Writing it in the way they do is the most general way I guess, because it's true no matter what bin size you end up choosing.

Obviously if any of this isn't clear let me know when we meet and I'll try to give a better explanation!

Best,

Henry



**From:** Reed, Alex <Alexander.Reed@warwick.ac.uk>  
**Sent:** 30 December 2019 16:34  
**To:** Charlesworth, Henry <H.Charlesworth@warwick.ac.uk>  
**Subject:** Meeting next term

Dear Dr Charlesworth,

I hope you are well and have had a pleasant Christmas!

Just wanted to get back to you about meeting next term to talk about FSM and its application to modelling (amongst other things) starling flocks. I have a few questions about the model itself and a few things I’m having trouble understanding (largely just minor confusions rather than any fundamental misunderstandings (I hope!)), but I mainly just wanted to meet and talk about your opinion on the strengths and drawbacks of the model/assumptions and in what ways the model can be expanded on in the future.

I also wanted to just give you a better understanding of my motivations – my project itself is a report on collective motion in animal groups (with an emphasis on starlings) and how various mathematical models/tools have been devised to simulate or quantify this kind of motion. In particular I am looking at two different approaches – the Cucker-Smale model (which has similar assumptions to the Vicsek model) as a dynamical systems/ODE approach and the FSM model as a more biologically realistic model based on decision making principles. As part of the latter I was hoping to include your personal thoughts as one of the creators of the model; with your consent I was hoping I could audio-record our talk with a Dictaphone in order to be able to reference it later.

Regarding actually meeting this term – I’m on campus this year so I can be around any time from next week. From Monday to Friday I finish lectures at 1pm, 1pm, 10am, 4pm and 4pm respectively but I can make myself free at any time if these are inconvenient. If it is easier to meet off campus I can also happily do that!

I’ve also attached the few questions I had about the terminology in case you’d rather answer those over email.

Thank you so much for your time and Happy New Year!

Kind regards,

Alex Reed

Attached questions were the following:

1. Is it possible that, at any given time, an agent can have a decision tree with more than one branch to choose from (i.e. two or more branches have the same maximal number of distinct visual states)? If so, how does it then decide which action to take?
2. In the ‘toy example’ given in the paper (and pasted below), action z1 leads to three nodes with four visual states – was this meant to read four nodes with four visual states or am I misunderstanding the model?

A close up of a person

Description automatically generated

1. Could you explain what you mean by “marginal opacity”? I’m confused as to whether this means an external observer sees the flock as being partially opaque or the agents themselves see their surroundings as being such.
2. Regarding the correlation length calculation as described in the SI document – I just wanted to confirm if this is the quantity given in “Scale-free correlations in starling flocks” (*Cavagna et al.*), i.e. the zero of the function

A picture containing object

Description automatically generated

Where c\_0 is the normalization factor, u\_i are velocities in centre of mass coordinates and δ is the Dirac delta function. The reference given in the SI is “Long-range order in a two-dimensional dynamical XY model” (*Toner & Yu*) but I couldn’t find the description of the correlation length in that paper.

If so, can I just confirm that δ takes the value 1 at r = r\_ij and 0 otherwise? I just wanted to check as I’m not used to the notation (I thought it took the value infinity at r = r\_ij but this wouldn’t make much sense)!